

Magnetic behaviour of dirty multiband superconductors near the upper critical field.

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Magnetic properties of dirty multiband superconductors near the upper critical field are studied. The parameter κ_2 characterizing magnetization slope is shown to have a significant temperature variation which is quite sensitive to the pairing interactions and relative strengths of intraband impurity scattering. In contrast to single-band superconductors the increase of κ_2 at low temperatures can be arbitrary large determined by the ratio of minimal and maximal diffusion coefficients in different bands. Temperature dependencies of $\kappa_2(T)$ in two-band MgB_2 and iron-based superconductors are shown to be much more sensitive to the multiband effects than the upper critical field $H_{c2}(T)$.

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I. INTRODUCTION

Recently a number of multiband superconductors have been discovered where the pairing of electrons is supposed to take place simultaneously in several bands overlapping at the Fermi level^{1–8}. One of the first such superconductors found was MgB_2 ¹ which has two distinct superconducting gaps residing on different sheets of the Fermi surface^{2,3,9}. Up to date MgB_2 has the highest critical temperature $T_c = 40$ K among simple binary compounds¹⁰. Later on multiband superconductivity has been established in iron-pnictides^{6–8}. There a strong interband interaction mediated by antiferromagnetic excitations has been suggested to play the dominant role in pairing resulting in the peculiar s_{\pm} symmetry of the order parameter. The highest T_c among iron-pnictides is above 100 K detected in atomically thin films of FeSe ¹¹.

Besides their high T_c both the two-gap MgB_2 and iron-based superconductors have remarkable magnetic properties. The possibility of type-1.5 superconductivity has been suggested to explain vortex clusterization detected in MgB_4 ^{12,13} and Sr_2RuO_4 ¹⁴. The observed unconventional vortex patterns can result from a non-monotonic vortex interaction generated by the interplay of multiple superconducting coherence lengths in multicomponent superconductors^{15–17}.

In pulsed field experiments the critical fields up to 60 T have been observed in iron pnictides^{18–23}. Such values of H_{c2} are high enough to reach a paramagnetic limit^{20,24–28} leading to the possibility of the Fulde-Ferrel-Larkin-Ovchinnikov transition²⁹. Comparably large upper critical fields were obtained by introducing disorder in MgB_2 thin films where the record values of $H_{c2}^{\perp} = 35$ T and $H_{c2}^{\parallel} = 51$ T were observed perpendicular and parallel to the crystal anisotropy ab plane respectively³⁰.

The interplay of several pairing channels in multiband superconductors was predicted to produce convex-shaped temperature dependencies of the upper critical field^{29,31–33}. Due to their anomalous shapes the $H_{c2}(T)$ curves reach much larger values at $T \rightarrow 0$

than was expected from a one-gap theory $H_{c2}(0) = 0.69T_c H_{c2}(T_c)$ ^{34,35}. This explains an enormous enhancement of the upper critical field in MgB_2 by non-magnetic impurities^{36–38}. Experiments measuring upper critical fields in the disordered MgB_2 ³⁰ and certain iron-based superconductors¹⁸ are consistent with theoretical calculations using a two-gap model^{29,31,32}. Therefore a convex shape of $H_{c2}(T)$ dependence is considered as one of the hallmarks of multiband pairing^{18,26,39–41}. However it is not a universal feature of multiband superconductors since concave $H_{c2}(T)$ curves were observed in MgB_2 without artificially introduced disorder¹⁰ as well as in many iron-pnictide compounds^{21,22,27,42}.

In order to find a robust test for multiband pairing it is natural to look for an unusual behaviour of magnetization at $H < H_{c2}$. In the vicinity of H_{c2} magnetization M_z can be characterized by the parameter $\kappa_2(T)$ introduced by Maki⁴³

$$M_z = \frac{H - H_{c2}}{4\pi\beta_L(2\kappa_2^2 - 1)}, \quad (1)$$

where z -axis is directed along the magnetic field, β_L is an Abrikosov parameter equal to 1.16 for a triangular lattice⁴⁴. In single-band superconductors the parameter κ_2 has been studied extensively in the clean⁴⁵ and dirty limits^{46,47}, for the arbitrary strength of impurity scattering^{48,49} and taking into account strong electron-phonon coupling effects⁵⁰. Dirty single-band superconductors were shown to have a universal behaviour characterized by a slow monotonic increase of κ_2 ^{43,46,47} with cooling from $\kappa_2(T = T_c) = \kappa_{GL}$ to $\kappa_2(T = 0) \approx 1.2\kappa_{GL}$, where κ_{GL} is the Ginzburg-Landau parameter at $T = T_c$. The theoretical calculations were found to be in good agreement with experimentally measured $\kappa_2(T)$ dependencies in several superconducting alloys.^{51–53}

The parameter κ_2 is a basic quantity of type-II superconductors determining their thermodynamic⁵⁴ and transport properties^{55,56} near H_{c2} . However the theory calculating κ_2 in multiband superconductors has been lacking. In the present paper we demonstrate that this parameter is much more sensitive to multiband effects

than the upper critical field. In MgB₂ and iron pnictides $\kappa_2(T)$ dependencies are shown to reveal pronounced signatures of multiband pairing in the regimes when $H_{c2}(T)$ curves deviate only slightly from the conventional single-band behaviour. The $\kappa_2(T)$ anomalies signal unconventional thermodynamic and transport characteristics of multiband superconductors.

The structure of this paper is as follows. In Sec.(II) basic equations of the multiband Usadel theory are introduced. General formulas describing the high-field magnetic response of dirty multiband superconductors are derived in Sec.(III) including equations for the H_{c2} in Sec.(III A) and the magnetization in Sec.(III B). Several examples of two-band superconductors are considered in Sec.(IV). Results are discussed in Sec.(V) and conclusions are given in Sec.(VI).

II. MULTIBAND USADEL THEORY.

We consider multiband superconductors in a dirty limit using the Usadel theory³¹. Each k -th band is described in terms of the quasiclassical Green's function matrix $\hat{g}_k = \hat{g}_k(\varepsilon, \mathbf{r})$ which is defined as follows

$$\hat{g}_k = \begin{pmatrix} g_k & f_k \\ -f_k^+ & -g_k \end{pmatrix} \quad (2)$$

and subject to the normalization constraint $\hat{g}_k^2 = 1$. The matrix Usadel equation reads^{31,57}

$$D_k \hat{\partial}_{\mathbf{r}} (\hat{g}_k \hat{\partial}_{\mathbf{r}} \hat{g}_k) - [\omega \tau_3 + \hat{\Delta}_k, \hat{g}_k] = 0 \quad (3)$$

where D_k is the diffusion constant, and $\hat{\Delta}_k(\mathbf{r}) = |\Delta_k| \tau_2 e^{-i\theta_k \tau_3}$ is the matrix gap function in k -th band. In Eq.(3) the covariant differential superoperator is defined by $\hat{\partial}_{\mathbf{r}} \hat{g} = \nabla \hat{g} - i e \mathbf{A} [\tau_3, \hat{g}]$. The 12 component of the matrix Eq.(3) yields:

$$\frac{D_k}{2i} (g_k \hat{\Pi}^2 f_k - f_k \nabla^2 g_k) = \Delta_k g_k - i \omega f_k \quad (4)$$

where $\hat{\Pi} = \nabla - 2ie\mathbf{A}$. Similar equation given by the 21 component of (3) yields $f^+(\mathbf{r}, \omega) = -f^*(\mathbf{r}, \omega)$. The gap in each band is determined by self-consistency equations

$$\Delta_k(\mathbf{r}) = 2i\pi T \sum_{j=1}^N \sum_{n=0}^{N_D} \lambda_{kj} f_j(\omega_n) \quad (5)$$

where $\hat{\lambda}$ is the $N \times N$ coupling matrix satisfying general symmetry relations $\nu_k \lambda_{kj} = \nu_j \lambda_{jk}$ and the sum by Matsubara frequencies $\omega_n = (2n+1)\pi T$ is taken in the limits $N_D(T) = \Omega_D/(2\pi T)$ set by the Debye frequency Ω_D . The electric current density is given by

$$\mathbf{j} = i\pi T \sum_{k=1}^N \sum_{n=0}^{\infty} \frac{\sigma_k}{e} \text{Tr}[\tau_3 \hat{g}_k(\omega_n) \hat{\partial}_{\mathbf{r}} \hat{g}_k(\omega_n)] \quad (6)$$

where the partial conductivities are $\sigma_k = 2e^2 \nu_k D_k$ and ν_k are the densities of states per one spin projection. The sum over frequencies in Eq.(6) converges therefore no cutoff is needed. The magnetization of a superconducting sample \mathbf{M} is determined by the current (6) according to the usual relation $\nabla \times \mathbf{M} = \mathbf{j}$.

III. MULTIBAND SUPERCONDUCTORS IN LARGE MAGNETIC FIELDS.

A. The upper critical field H_{c2} .

At large magnetic fields $H_{c2} - H \ll H_{c2}$ we can apply approximations related to the smallness of the order parameter $|\Delta_k| \propto \sqrt{1 - H/H_{c2}}$. To calculate the structure of a vortex lattice in a two-band superconductor let us consider the linear integral-differential system consisting of Usadel Eqs.(4) linearized with respect to the normal state solution

$$\hat{L}_{\omega} f_k = i \Delta_k; \quad \hat{L}_{\omega} = \frac{D_k}{2} \hat{\Pi}_0^2 - |\omega|, \quad (7)$$

supplemented by the self-consistency relation (5). In a linearised theory the magnetic field is not perturbed by the vortex currents therefore we put $\mathbf{B}_0 = H_{c2} \mathbf{z}$ and choose a Landau gauge in Eq.(7) $\mathbf{A}_0 = H_{c2} x \mathbf{y}$. Then the gradient term in Eq.(7) is $\hat{\Pi}_0 = \nabla - 2ie\mathbf{A}_0$. A periodic vortex lattice is described by the Abrikosov solution of Eqs.(7,5) which in general has the following form

$$\Delta_k(\mathbf{r}) = \Delta b_k \Psi(\mathbf{r}) \quad (8)$$

$$\Psi(\mathbf{r}) = \sum_n C_n e^{i n p y} \Psi_0(x - n x_0) \quad (9)$$

where $|C_n| = 1$, $x_0 = p/(2eH_{c2})$ and parameter p is determined by the lattice geometry. The lowest Landau level wave function $\Psi_0(x) = 2L_H \sqrt{\pi} \exp(-x^2/2L_H^2)$ satisfies $(L_H^2 \partial_x^2 - x^2/L_H^2 + 1)\Psi_0 = 0$ where the magnetic length is $L_H = 1/\sqrt{2eH_{c2}}$. The gaps Δ_k are determined by the common amplitude Δ and a normalized set of components $\sum_k b_k^2 = 1$.

The solution of Eq. (7) yields

$$f_k(\mathbf{r}, \omega_n) = \frac{\Delta_k(\mathbf{r})}{i(q_k + |\omega_n|)} \quad (10)$$

where $q_k = eH_{c2}D_k$. Substituting the ansatz (8,10) to the self-consistency Eq.(5) we get the homogeneous linear system $\hat{A}(b_1, \dots, b_N)^T = 0$ for the order parameter amplitudes where

$$\hat{A} = \hat{A}^{-1} - \hat{I} [G_0 + \ln(T_c/T) + \psi(1/2) - \psi(1/2 + \hat{\rho})]. \quad (11)$$

Here $\psi(x)$ is a di-gamma function and the diagonal matrix $\hat{\rho}$ is given by $(\hat{\rho})_{ij} = \delta_{ij} q_i/(2\pi T)$. The solvability condition $\det \hat{A} = 0$ determines the upper critical field H_{c2} of a dirty multiband superconductor.

It is instructive to consider in more detail Eq.(11) in two-band superconductors. In this case $G_0 = (\text{Tr } \Lambda - \lambda_0)/w$ where $w = \lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21}$, $\lambda_0 = \sqrt{\lambda_-^2 + 4\lambda_{12}\lambda_{21}}$ and $\lambda_- = \lambda_{11} - \lambda_{22}$. The equation $\det \hat{A} = 0$ can be resolved in terms of the $\ln(T/T_c)$ yielding in general two different solutions

$$\ln(T/T_c) = -(U_1 + U_2 + \lambda_0/w)/2 \quad (12)$$

$$+ [(U_1 - U_2 - \lambda_-/w)^2/4 + \lambda_{12}\lambda_{21}/w^2]^{1/2}$$

$$\ln(T/T_c) = -(U_1 + U_2 + \lambda_0/w)/2 \quad (13)$$

$$- [(U_1 - U_2 - \lambda_-/w)^2/4 + \lambda_{12}\lambda_{21}/w^2]^{1/2},$$

where $U_k = \psi(1/2 + \rho_k) - \psi(1/2)$. Taking the limit $T \rightarrow T_c$ one can see that the physical solutions are (i) (12) in case when $w \equiv \det \hat{\Lambda} > 0$ and (ii) (13) in case when $w \equiv \det \hat{\Lambda} < 0$. While the case (i) corresponds to the coupling parameters of MgB₂^{31,32} the case (ii) describes multiband superconductors with interband-dominated pairing when $\lambda_{12}\lambda_{21} > \lambda_{11}\lambda_{22}$ such as iron-pnictide compounds^{6,8,18}.

B. The magnetization slope dM_z/dH .

Magnetic field created by vortex currents (6) can be found using the solution (8). Taking into account that Green's functions $f_k(\mathbf{r})$ given by (9) satisfy the relation

$$i\partial_x f_k = (\partial_y - 2ieA_y)f_k \quad (14)$$

we obtain the multi-band expression for magnetization

$$4\pi M_z(\mathbf{r}) = - \sum_k \frac{\sigma_k}{eT} \psi'_k |\Delta_k|^2. \quad (15)$$

The order parameter amplitudes Δ_k can be found according to the following straightforward algorithm. First, non-linear corrections f_k are obtained from Eq.(4) taking into account higher-order terms in Δ_k :

$$\hat{L}_\omega \tilde{f}_k = - \frac{i\Delta_k |\Delta_k|^2}{2(q_k + |\omega|)^2} + \quad (16)$$

$$\frac{eD_k \{\hat{\Pi}_0, \mathbf{A}_1\} \Delta_k}{(q_k + |\omega|)} - \frac{i(2q_k \Delta_k |\Delta_k|^2 + D_k \Delta_k \nabla^2 |\Delta_k|^2)}{4(q_k + |\omega|)^3}.$$

Then the self-consistency Eq.(5) yields a non-homogeneous linear system for the corrections $\tilde{\Delta}_j$:

$$\hat{C}_{kj} \tilde{\Delta}_j = 2\pi i T \sum_{n=0}^{\infty} \nu_k \tilde{f}_k(\omega_n), \quad (17)$$

$$\hat{C} = \hat{\nu} \hat{\Lambda}^{-1} - 2\pi i T \sum_{n=0}^{N_D} \hat{\nu} \hat{L}_{\omega_n}^{-1}, \quad (18)$$

where $\hat{\nu}_{kj} = \nu_k \delta_{kj}$. Since the matrix $\hat{\nu} \hat{\Lambda}^{-1}$ is symmetric the operator \hat{C} is hermitian and the linear solution

(8) belongs to its kernel $\hat{C}_{kj} \Delta_j = 0$. Hence multiplying the l.h.s. of a non-homogeneous Eq.(17) by Δ_k^* we get $\sum_{k,j} \langle \Delta_k^* \hat{C}_{kj} \tilde{\Delta}_j \rangle = 0$. Thus Eq.(17) is solvable if its r.h.s. is orthogonal to the linear solution

$$\sum_k \sum_{n \geq 0} \nu_k \langle \Delta_k^* \tilde{f}_k(\omega_n) \rangle = 0. \quad (19)$$

To calculate each term in the sum (19) we multiply Eq.(16) by Δ_k^* and average over space coordinates taking into account the relations

$$\langle \Delta_k^* \hat{L}_\omega \tilde{f}_k \rangle = -(|\omega| + q_k) \langle \Delta_k^* \tilde{f}_k \rangle \quad (20)$$

$$\langle \Delta_k^* \{\hat{\Pi}_0, \mathbf{A}_1\} \Delta_k \rangle = -i \langle B_1 |\Delta_k|^2 \rangle \quad (21)$$

$$2q_k \langle |\Delta_k|^4 \rangle + D_k \langle |\Delta_k|^2 \nabla^2 |\Delta_k|^2 \rangle = 0 \quad (22)$$

where $B_1 = -\delta H + 4\pi M_z$ and $\delta H = H_{c2} - H$. The relations (20,21) can be obtained by a straightforward calculation while (22) is less trivial although it has been used in the theory of single-band superconductors⁵⁸. The detailed derivation of Eq.(22) is shown in the Appendix (A).

To simplify the further derivation let us consider from the beginning a high- κ limit when $\sigma_k D_k \ll 1$. In this case we can neglect the magnetization in Eq.(22) to get finally

$$2i\pi^2 T^3 \sum_{n \geq 0} \langle \Delta_k^* \tilde{f}_k(\omega_n) \rangle = eT D_k \psi'_k \delta H \langle |\Delta_k|^2 \rangle - 2\sigma_k D_k \psi_k'^2 \tilde{\kappa}_k^2 \langle |\Delta_k|^4 \rangle, \quad (23)$$

where $\tilde{\kappa}_k$ are single-band parameters given by^{43,46}

$$\tilde{\kappa}_k = \left[\frac{-\psi_k''}{16\pi \sigma_k D_k \psi_k'^2} \right]^{1/2}. \quad (24)$$

Combining Eqs.(19) and (23) we obtain the order parameter amplitude in Eq.(8) given by

$$\Delta = \left[\frac{eT \delta H}{2\beta_L} \frac{\sum_k \nu_k b_k^2 D_k \psi'_k}{\sum_k \nu_k b_k^4 \sigma_k D_k \psi_k'^2 \tilde{\kappa}_k^2} \right]^{1/2}, \quad (25)$$

where the Abrikosov parameter is $\beta_L = \langle |\Psi|^4 \rangle / \langle |\Psi|^2 \rangle^2$.

The derived amplitude Δ is a basic parameter for calculations of thermodynamic and transport properties of superconductors near H_{c2} . In particular using Eq.(15) we obtain an expression for the space-averaged magnetization $M_z = -\delta H (dM_z/dH)$ where the slope is given by

$$\frac{dM_z}{dH} = \frac{(\sum_k \sigma_k b_k^2 \psi'_k) (\sum_k \nu_k b_k^2 D_k \psi'_k)}{8\pi \beta_L \sum_k \nu_k b_k^4 \sigma_k D_k \psi_k'^2 \tilde{\kappa}_k^2}. \quad (26)$$

Comparing Eq.(26) with the conventional parametrization (1) in the limit $\kappa_2 \gg 1$ we find an effective parameter

$$\kappa_2 = \sqrt{\frac{\sum_k \nu_k b_k^4 \sigma_k D_k \psi_k'^2 \tilde{\kappa}_k^2}{(\sum_k \sigma_k b_k^2 \psi'_k) (\sum_k \nu_k b_k^2 D_k \psi'_k)}}. \quad (27)$$

Close to the critical temperature κ_2 reduces to the Ginzburg-Landau parameter $\kappa_2(T_c) = \kappa_{GL} \equiv \lambda_L/\xi$ where λ_L is the London penetration length and $\xi = 1/\sqrt{2eH_{c2}}$ is the coherence length. Note that this definition of coherence length is applicable only for dense vortex lattices near H_{c2} . The physics of dilute vortex configurations in multicomponent systems is regulated by the interplay of multiple coherence lengths resulting in non-monotonic vortex interactions^{15–17}.

IV. EXAMPLES OF TWO-BAND SUPERCONDUCTORS.

The general formalism developed in previous sections can be applied to study magnetic properties of particular multiband compounds. To begin with we consider a two-band model of MgB_2 characterized by the coupling parameters⁵⁹ $\lambda_{11} = 0.81$, $\lambda_{22} = 0.285$, $\lambda_{12} = 0.119$, $\lambda_{21} = 0.09$. Temperature dependencies of $H_{c2}(T)$ and $\kappa_2(T)$ are shown in Fig.(1) for different values of (a) $D_1/D_2 = 1$; 0.5; 0.25, (b) $D_1/D_2 = 20$, (c) $D_1/D_2 = 0.05$. For equal diffusion coefficients $D_1/D_2 = 1$ the single-band behaviours^{34,35,46} of H_{c2} and κ_2 are recovered, see Fig.(1)a,b. As will be shown below this result is valid for any number of bands and arbitrary pairing matrix.

The disparity of diffusion coefficients $D_1/D_2 \neq 1$ results in significant variations of κ_2 . Comparing Figs.(1)a and (1)b one can see that $\kappa_2(T)$ is much more sensitive to the ratio of diffusivities than the second critical field. The curvature variations of $H_{c2}(T)$ are noticeable only in the limit $D_2 \ll D_1$ as shown Fig.(1c). As demonstrated in the Fig.(1e) the opposite limit $D_1 \ll D_2$ yields ordinary concave curves $H_{c2}(T)$ almost within the entire temperature domain except of the small vicinity of T_c . On the contrary temperature dependencies of $\kappa_2(T)$ shown for the same parameters in Fig.(1f) are drastically different from the single-band case. Of particular interest is a sharp increase of $\kappa_2(T \rightarrow 0)$ which is most pronounced under the condition $D_2 \ll D_1$ relevant to MgB_2 ³¹ [see Fig.(1d)]. Physically thus means that the slope of magnetization curve becomes much less steep as shown in the Fig.(3). The low-temperature increase of κ_2 can be considered as a feasible probe of the multiband pairing.

Next we consider two-band superconductors with pairing from interband repulsion $\lambda_{ii} = 0$ and $\lambda_{12} = \lambda_{21} = -0.5$ resulting in the s_{\pm} superconducting state^{6,8}. Such a model has been used to describe an unconventional convex behaviour of $H_{c2}(T)$ observed experimentally in iron-based compounds.¹⁸ Here we suggest that an independent and more sensitive test for the multiband physics in iron pnictides can be implemented by measuring temperature dependencies $\kappa_2(T)$. As can be seen in Fig.(2b) κ_2 demonstrate a sharp increase at low temperatures even for not too small values of the ratio D_1/D_2 and deviate strongly from the single-band behaviour shown by the green down-most curve. For the same parameters $H_{c2}(T)$

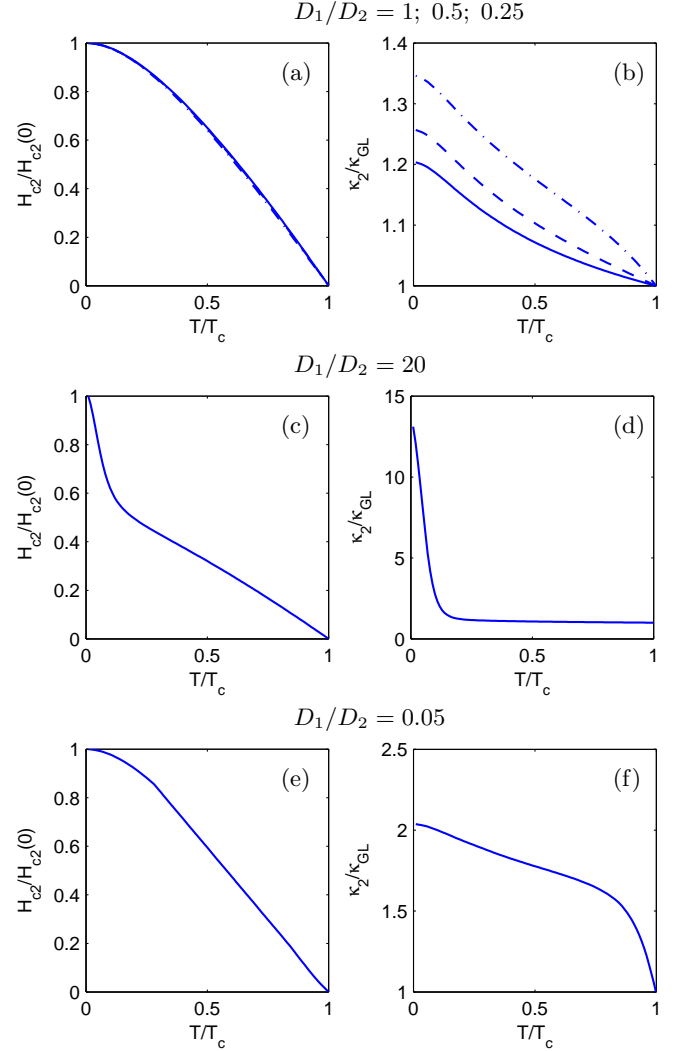


FIG. 1: (Color online) Magnetic properties of the two-band superconductor MgB_2 with coupling parameters mentioned in the text. The panels show (a,c,e) $H_{c2}(T)$ and (b,d,f) $\kappa_2(T)$ as given by Eqs.(12,27) for different values of the ratio D_1/D_2 . In (a,b) solid, dashed and dash-dotted lines correspond to $D_1/D_2 = 1$; 0.5; 0.25 respectively. In (a) these curves are almost undistinguishable. (c,d) $D_1/D_2 = 20$ and (e,f) $D_1/D_2 = 0.05$.

dependencies have only tiny deviations from the single-band one shown by the green up-most line in Fig.(2a).

Summarizing the above examples one can see that even a moderate disparity of diffusion constants when $D_1/D_2 \sim 1$ in two-band superconductors results in a significant increase of $\kappa_2(T)$ at low temperatures as compared to its value at the critical temperature $\kappa_2(T_c) = \kappa_{GL}$. In result the magnetization slope dM_z/dH becomes much less steep as compared to single-band superconductors. This behaviour is illustrated in Fig.(3) which shows the slopes dM_z/dH normalized to their values at T_c as functions of temperature for different values of D_1/D_2 . In both (a) and (b) panels the up-most green curve shows

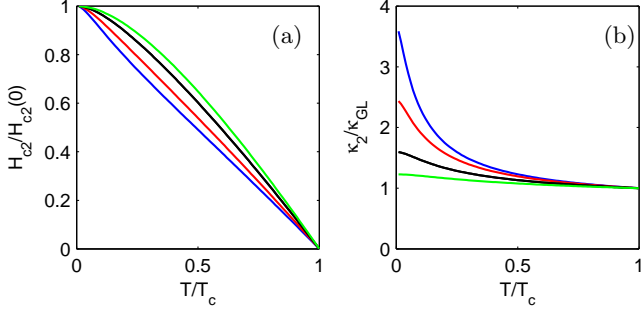


FIG. 2: (Color online) Magnetic properties of a two-band superconductor with interband- dominated pairing $\lambda_{11} = \lambda_{22} = 0$, $\lambda_{12} = \lambda_{21} = -0.5$ corresponding to iron-pnictide superconductors. (a) $H_{c2}(T)$ curves from *top to bottom* correspond to $D_1/D_2 = 1; 0.25; 0.1; 0.05$. (b) The magnetization parameter $\kappa_2(T)$ as given by the Eq.(27). The curves from *bottom to top* correspond to the same sequence of D_1/D_2 as in (a).

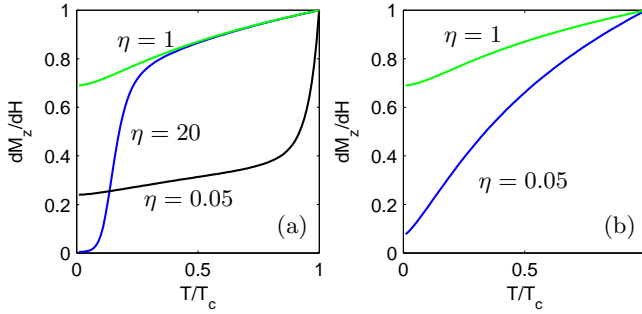


FIG. 3: (Color online) Slopes of the magnetization curves dM_z/dH in two-band superconductors with coupling parameters corresponding to (a) MgB_2 $\lambda_{11} = 0.81$, $\lambda_{22} = 0.285$, $\lambda_{12} = 0.119$, $\lambda_{21} = 0.09$ and (b) iron-pnictides $\lambda_{11} = \lambda_{22} = 0$, $\lambda_{12} = \lambda_{21} = -0.5$. The value of $\eta = D_1/D_2$ is marked near each curve. The slopes are normalized to their values at T_c .

a single-band behaviour which is reproduced universally for $D_1 = D_2$ irrespective of the coupling parameters. The changes in magnetization slopes can be directly measured and yield an important information about multi-band pairing and diffusion constants in different bands.

V. DISCUSSION.

The convex shape of $H_{c2}(T)$ curves is often considered as a signature of multiband pairing^{18,39,41}. However as demonstrated by the above two-band examples the conditions for having pronounced convexity such as shown in Fig.(1c) are quite restrictive. If the disparity of diffusivities is not extreme $D_1/D_2 \sim 1$ then deviations of $H_{c2}(T)$ from the conventional single-band theory are not significant. However even in this case it is possible to detect signatures of multiband pairing in the magnetic response measuring the magnetization slope at high fields. Shown

in the right columns of Figs.(1,2) $\kappa_2(T)$ dependencies are quite sensitive to variations of diffusivities even in the range of parameters when $H_{c2}(T)$ curves look almost the same as the single-band one.

To understand qualitative features of $\kappa_2(T)$ in multi-band superconductors it is instructive to consider several characteristic cases. First let us recover single-band results for H_{c2} and κ_2 assuming all diffusivities to be equal $D_k = D$ for $k = 1 \dots N$. In this case $\rho_k = \rho$ so that Eq.(11) reduces to $\hat{A} = \hat{A}^{-1} - \hat{I}[G_0 - U(\rho) - \ln(T/T_c)]$. The solvability condition $\det \hat{A} = 0$ yields a single-band equation for the upper critical field^{34,35} $U(\rho) + \ln(T/T_c) = 0$. The corresponding eigen vector is temperature independent and determined by the equation $(\hat{A}^{-1} - \hat{I}G_0)\mathbf{b} = 0$. Then taking into account that $\kappa_2(T_c) = \kappa_{GL}$ from Eq.(27) we obtain the analytical expression $\kappa_2^2/\kappa_{GL}^2 = -\pi^4\psi''/[56\zeta(3)\psi'^2]$ coinciding with the single-band result⁴⁶.

To explain significant variations of κ_2 let us compare a low- and high-temperature asymptotic of the Eq.(27):

$$\kappa_2(T_c) = \sqrt{\frac{7\zeta(3)}{2\pi^5 e^2} \frac{\sum_k \nu_k b_k^4}{(\sum_k \nu_k D_k b_k^2)^2}}, \quad (28)$$

$$\kappa_2(0) = \sqrt{\frac{1}{32\pi e^2} \frac{\sum_k \nu_k b_k^4 D_k^{-2}}{(\sum_k \nu_k b_k^2)^2}}, \quad (29)$$

where we have used that $\psi'_k \approx \rho_k^{-1}$, $\psi''_k \approx -\rho_k^{-2}$ at $T \rightarrow 0$ and $\psi'_k = \pi^2/2$, $\psi''_k = -14\zeta(3)$ at $T = T_c$.

Eqs.(28,29) demonstrate that in the limit of a strong disparity between diffusivities the value of $\kappa_2(T_c)$ is determined by the maximal diffusivity while $\kappa_2(0)$ is determined by the minimal one. Therefore $\kappa_{GL} = \kappa_2(T_c) \sim 1/D_1$ and $\kappa_2(0) \sim 1/D_2$ so that the low temperature increase of $\kappa_2(0)/\kappa_{GL} \sim D_1/D_2 \gg 1$ is determined by the ratio of maximal and minimal diffusivities $D_1 = \max(D_k)$ and $D_2 = \min(D_k)$ respectively.

Such a behaviour can be qualitatively understood as follows. Near the critical temperature Eq. (11) reduces to $\hat{A} = \hat{A}^{-1} - \hat{I}G_0$ so that gap amplitudes b_k are determined only by the coupling matrix. The magnetic field is small so that $\rho_k \ll 1$ and its influence on the gap amplitudes is negligible. However the contributions to superconducting current and magnetization (6,15) from each band are proportional to the corresponding diffusion coefficients. Hence in the limit of strong disparity $D_1/D_2 \ll (\gg) 1$ the band with the largest diffusivity provides a dominant contribution to the magnetization near T_c . On the other hand at low temperatures the magnetic field is large so that $\rho_k \gg 1$ and therefore can effectively suppress superconducting correlations. From Eq.(10) one can see that the anomalous function amplitude is smaller in bands with larger diffusivities. Hence at $T \rightarrow 0$ the most significant contribution to the magnetic response and κ_2 is determined by the band with the smallest diffusivity.

Finally, the calculations presented in this paper consider only the orbital depairing mechanism and neglect

paramagnetic effects which are important in iron pnictide compounds with large critical fields^{24–28}. High values of $H_{c2} = \phi_0/(2\pi\xi^2)$ in these materials are determined by short coherence lengths $\xi \sim 1 - 3$ nm²⁹ which are not consistent with the dirty limit approximation considered here. It is possible however to develop a theory for κ_2 in superconductors with arbitrary impurity concentration^{48,49}. In single-band superconductors $\kappa_2(T \rightarrow 0)$ diverges in the clean limit but for experimentally relevant finite impurity concentrations the changes are not dramatic as compared to the dirty limit⁵¹. On the other hand in the multiband case an interplay between different Fermi velocities and impurity scattering rates should result in a non-trivial modifications of $\kappa_2(T)$ temperature dependencies.

VI. CONCLUSION

To conclude we have calculated the parameter κ_2 characterizing magnetization slopes dM_z/dH in dirty multiband superconductors at high fields $H_{c2} - H \ll H_{c2}$. The developed theory describes any number of superconducting bands and arbitrary set of pairing constants. We have shown quite generally that in contrast to the dirty single-band superconductors the temperature dependencies of $\kappa_2(T)$ have remarkable features which are highly sensitive to the multiband effects. The low-temperature increase of κ_2 as compared to its value at T_c is found to be strongly pronounced even for the moderate disparity of diffusion coefficients in different bands. This effect should be particularly appealing for experimental identification since it could unambiguously confirm unconventional magnetic behaviour of multiband superconductors. We have considered several examples of two-band materials like MgB₂ and iron pnictides and demonstrated that κ_2 is much more sensitive than H_{c2} to the ratio of diffusion coefficients in different bands. The established relations between κ_2 and gap function amplitudes provide a basis to study thermodynamic and transport properties of multiband superconductors in high magnetic fields.

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Appendix A: Proof of the relation Eq.(22)

We use the relations $\nabla = (\partial_+ + \partial_-)/2$ introducing the operators $\partial_{\pm} = \mathbf{x}\partial_x + \mathbf{y}(\partial_y \pm 2ieA_y)$ so that $\partial_{\pm}^2 = \partial_x^2 + (\partial_y \pm 2ieA_y)^2$. The gap functions satisfy

$$\partial_+^2 \Delta^* = -2eH_{c2} \Delta^* \quad (\text{A1})$$

$$\partial_-^2 \Delta = -2eH_{c2} \Delta \quad (\text{A2})$$

Due to the relations

$$\partial_x \Delta = i(\partial_y - 2ieA_y) \Delta \quad (\text{A3})$$

$$\partial_x \Delta^* = -i(\partial_y + 2ieA_y) \Delta^* \quad (\text{A4})$$

we get

$$(\partial_- \Delta)^2 = 0; \quad (\partial_+ \Delta^*)^2 = 0 \quad (\text{A5})$$

Then the average is given by

$$4\langle |\Delta|^2 \nabla^2 |\Delta|^2 \rangle = \langle |\Delta|^2 (\partial_+^2 + \partial_-^2 + 2\partial_+ \partial_-) |\Delta|^2 \rangle.$$

Let us consider the three terms in separate

$$\text{(i)} \langle |\Delta|^2 \partial_+^2 |\Delta|^2 \rangle = \quad (\text{A6})$$

$$\begin{aligned} & \langle |\Delta|^2 (\Delta^* \partial_+^2 \Delta + \Delta \partial_+^2 \Delta^* + 2\partial_+ \Delta \partial_+ \Delta^*) \rangle = \\ & -2eH_{c2} \langle |\Delta|^4 \rangle + \langle |\Delta|^2 \Delta^* \partial_+^2 \Delta \rangle + 2\langle |\Delta|^2 \partial_+ \Delta \partial_+ \Delta^* \rangle \end{aligned}$$

$$\text{(ii)} \langle |\Delta|^2 \partial_-^2 |\Delta|^2 \rangle = \quad (\text{A7})$$

$$\begin{aligned} & \langle |\Delta|^2 (\Delta^* \partial_-^2 \Delta + \Delta \partial_-^2 \Delta^* + 2\partial_- \Delta \partial_- \Delta^*) \rangle = \\ & -2eH_{c2} \langle |\Delta|^4 \rangle + \langle |\Delta|^2 \Delta \partial_-^2 \Delta^* \rangle + 2\langle |\Delta|^2 \partial_- \Delta \partial_- \Delta^* \rangle \end{aligned}$$

$$\text{(iii)} -2\langle |\Delta|^2 \partial_+ \partial_- |\Delta|^2 \rangle = \quad (\text{A8})$$

$$\begin{aligned} & \langle (\partial_+ |\Delta|^2)^2 \rangle + \langle (\partial_- |\Delta|^2)^2 \rangle = \\ & \langle (\partial_+ \Delta)^2 \Delta^{*2} \rangle + \langle (\partial_+ \Delta^*)^2 \Delta^2 \rangle + 2\langle |\Delta|^2 (\partial_+ \Delta)(\partial_+ \Delta^*) \rangle + \\ & \langle (\partial_- \Delta)^2 \Delta^{*2} \rangle + \langle (\partial_- \Delta^*)^2 \Delta^2 \rangle + 2\langle |\Delta|^2 (\partial_- \Delta)(\partial_- \Delta^*) \rangle = \\ & \langle (\partial_+ \Delta)^2 \Delta^{*2} \rangle + 2\langle |\Delta|^2 (\partial_+ \Delta)(\partial_+ \Delta^*) \rangle + \\ & \langle (\partial_- \Delta^*)^2 \Delta^2 \rangle + 2\langle |\Delta|^2 (\partial_- \Delta)(\partial_- \Delta^*) \rangle \end{aligned}$$

where we took into account the Eqs.(A5). Collecting all terms we get

$$4\langle |\Delta|^2 \nabla^2 |\Delta|^2 \rangle = -4eH_{c2} \langle |\Delta|^4 \rangle + \quad (\text{A9})$$

$$\langle \Delta^{*2} [\Delta \partial_+^2 \Delta - (\partial_+ \Delta)^2] \rangle + \langle \Delta^2 [\Delta^* \partial_-^2 \Delta^* - (\partial_- \Delta^*)^2] \rangle$$

The last two terms here can be transformed in a similar way as follows

$$\Delta \partial_+^2 \Delta - (\partial_+ \Delta)^2 = \quad (\text{A10})$$

$$\Delta \partial_-^2 \Delta - (\partial_- \Delta)^2 = -4eH_{c2} \Delta^2$$

$$\Delta^* \partial_-^2 \Delta^* - (\partial_- \Delta^*)^2 = \quad (\text{A11})$$

$$\Delta^* \partial_+^2 \Delta^* - (\partial_+ \Delta^*)^2 = -4eH_{c2} \Delta^{*2}$$

so that finally we get

$$\langle |\Delta|^2 \nabla^2 |\Delta|^2 \rangle = -2eH_{c2} \langle |\Delta|^4 \rangle$$

which proves the Eq.(22).

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